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Fashion and Business Cycles with Snobs and Bandwagoners in a Multi-Sector Growth Model[1]

Prof. Wei-Bin Zhang [a*]

[a] Ritsumeikan Asia Pacific University, Japan.
[*]Corresponding author's email address: wbz1@apu.ac.jp

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ABSTRACT

The purpose of this study is to examine business cycles in the dynamic growth model with interactions between fashion, economic growth and income and wealth distribution by Zhang (2016). There few theoretical models of fashion with wealth accumulation and preference change on the basis of microeconomic foundation. This study introduces fashion into neoclassical growth theory. This study generalizes Zhang's model by making all the time-independent parameters as time-dependent parameters. We simulate the motion of the economic system. We carry out comparative dynamic analysis with regard to periodic perturbations in some parameters. We show how exogenous period changes in these parameters lead to business cycles.

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1. Introduction

It is well known that business cycles theories use different determinants to identify economic oscillations in different economic systems. Nevertheless, there are a few economic models which exhibit periodic changes due to exogenous periodic changes with fashions. "Fashion is important because it is in almost everything." (Daniels, 1951). Fashion industry has special economic features. Miller *et al*. (1993) summarize types of models in theory of fashion as follows: (i) external-individual models - aesthetic perceptions and learning model; (ii) external-social models – social conflict model, art movement model, ideas of beauty model, mass market model, market infrastructure model; (iii) internal-individual models – demand model, scarcity-rarity model, conspicuous consumption model, individualism-centered model, conformity-centered model, uniqueness motivation model; (iv) internal-social models – trickle-down theory, collective behavior theory, adoption and diffusion model, symbolic communication model, subcultural leadership, model, spatial diffusion model, historical resurrection model, and historical continuity model. Hemphill and Suk (2009: 1148) describe dynamics of fashion as follows: "People flock to ideas, styles, methods, and practices that seem new and exciting, and then eventually the intensity of that collective fascination subsides, when the newer and hence more exciting emerge on the scene. Participants of social practices that value innovation are driven to partake of what is "original," "cutting edge," "fresh," "leading," or "hot." But with time, those qualities are attributed to others, and another trend takes

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shape." There are many empirical studies on fashions. For instance, Barber (1999: 459) summarizes the research on cycles of bodily attractiveness as follows: "Generalizing from cycles of bodily attractiveness for women, it was hypothesized that dress styles are reflective of reproductive economics. Using data from three studies of dress fashion extending from 1885 to 1976, the prediction was tested that short skirts (signaling sexual accessibility) would be correlated with low sex ratios (indicating limited marital opportunity for women), with increased economic opportunities for women and with marital instability." According to Hemphill and Suk (2009: 1148), fashion industry "is the major output of a global business with annual U.S. sales of more than $200 billion – larger than those of books, movies, and music combined." Nevertheless, there are only a few formal economic models of fashion within a framework of economic growth (e.g., Pollak, 1976; Stigler and Becker, 1977; Abel, 1990; Karni and Schmeidler, 1990; Bianchi, 2002; Nakayama and Nakamura, 2004; Caulkins, *et al.*, 2007; Zhou, *et al.*, 2015).

The purpose of this study is to make a unique contribution to business cycles theories by demonstrating business cycles in an economic growth model with fashion. We introduce exogenous changes into a model with interactions between fashion, economic growth and income and wealth distribution. The model proposed by Zhang (2016) is the first formal model in neoclassical growth theory with fashion and heterogeneous households. This study generalizes Zhang's model by allowing all the time-independent parameters to be time dependent. We specially show how business cycles in the fashion industry can be generated by different exogenous periodic shocks. The rest of the paper is organized as follows. Section 2 generalizes Zhang's model by allowing all the time-independent parameters to be time-dependent. Section 3 examines dynamic properties of the model and simulates the model with three groups. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study.

2. The basic model

This section is to generalize Zhang's growth model with fashion by allowing all the time-independent parameters to be time-dependent (Zhang, 2016). The generalization allows us to analyze time-dependent shocks. The economy has three sectors, called respectively fashion, capital good, and consumer good sectors. The modelling of capital good and consumer good sectors is based on the Uzawa two sector model (Uzawa, 1961; Burmeister and Dobell 1970; and Barro and Sala-i-Martin, 1995). The fashion sector supplies fashion goods. Fashion goods are instantly consumed goods. Capital depreciates at an exponential rate $\delta_k(t)$ ($0 < \delta_k < 1$).. All markets are perfectly competitive. Saving is undertaken only by households and assets of the economy are owned by the households. Factors are fully utilized at every moment. All earnings of firms are distributed in the form of payments to factors of production and labor. In addition to the snob and the bandwagoner as in Giovinazzo and Naimzada (2015), we classify the population into the snob, the bandwagoner, and common consumer. Common consumers have no interest in fashion. The population of consumer, snob, and bandwagoner are respectively $\bar{N}_j(t)$, $j = C, S, B$. Prices are measured in terms of capital good and the price of the commodity is unity. Let $w_j(t)$ and $r(t)$ represent respectively the wage rates and rate of interest. We use $p_C(t)$ and $p_F(t)$ to stand for prices of consumer and fashion goods, respectively. The total capital stock $K(t)$ is allocated between the three sectors. We use subscript index, i, s, and f to stand for capital, consumer, and fashion good sectors, respectively. Let $N_m(t)$ and $K_m(t)$ stand for the labor force and capital stocks employed by sector m. We use $F_m(t)$ to stand for the production function of sector m, $m = i, s, f$. The total population $\bar{N}(t)$ and total qualified labor supply $N(t)$ are

$$\bar{N}(t) = \bar{N}_C(t) + \bar{N}_S(t) + \bar{N}_B(t), \quad N(t) = h_C(t)\bar{N}_C(t) + h_S(t)\bar{N}_S(t) + h_B(t)\bar{N}_B(t), \tag{1}$$

in which $h_j(t)$ is the human capital of group j. The labor force is fully employed

$$N_i(t) + N_s(t) + N_f(t) = N(t). \tag{2}$$

2.1 The capital good sector

The production function of capital good sector is

$$F_i(t) = A_i(t)K_i^{\alpha_i(t)}(t)N_i^{\beta_i(t)}(t), \quad \alpha_i(t), \beta_i(t) > 0, \quad \alpha_i(t) + \beta_i(t) = 1, \tag{3}$$

where $A_i(t)$, $\alpha_i(t)$, and $\beta_i(t)$ are positive parameters. The marginal conditions for the capital good sector imply

$$r(t) + \delta_k(t) = \frac{\alpha_i(t) F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i(t) F_i(t)}{N_i(t)}, \tag{4}$$

where $w(t)$ is the wage rate of per qualified labor input. The wage rate of group j is

$$w_j(t) = h_j(t) w(t). \tag{5}$$

2.2 Consumer good and fashion good sectors

The production functions of the two sectors are

$$F_m(t) = A_m(t) K_m^{\alpha_m(t)}(t) N_m^{\beta_m(t)}(t), \quad \alpha_m(t) + \beta_m(t) = 1, \quad \alpha_m(t), \beta_m(t) > 0, \quad m = s, f, \tag{6}$$

where $A_m(t)$, $\alpha_m(t)$, and $\beta_m(t)$ are the technological parameters of the two sectors. The marginal conditions are

$$r(t) + \delta_k(t) = \frac{\alpha_m(t) p_m(t) F_m(t)}{K_m(t)}, \quad w(t) = \frac{\beta_m(t) p_m(t) F_m(t)}{N_m(t)}. \tag{7}$$

2.3 Current income and disposable income

We use $\bar{k}_j(t)$ to stand for per capita wealth of group j, $\bar{k}_j(t) = \bar{K}_j(t) / \bar{N}_j(t)$ where $\bar{K}_j(t)$ is the total wealth held by group j. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$ and the wage income is

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t).$$

The per capita disposable income $\hat{y}_j(t)$ is the sum of the current disposable income and the value of wealth

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{8}$$

2.4 Budget constraints with fashion good

The representative household of group j distributes the total available budget between savings $s_j(t)$, consumer good $c_j(t)$, and fashion good $f_j(t)$ ($f_C(t) = 0$).The budget constraints are

$$p_C(t) c_j(t) + s_j(t) + p_F(t) \bar{f}_j(t) = \hat{y}_j(t). \tag{9}$$

2.5 Utility functions with fashion

Utility level $U_j(t)$ is dependent on $c_j(t)$, $s_j(t)$ and $f_j(t)$ as follows

$$U_j(t) = c_j^{\xi_{j0}(t)}(t) s_j^{\lambda_{j0}(t)}(t) f_j^{\theta_{j0}(t)}(t), \quad \xi_{j0}(t), \lambda_{j0}(t) > 0, \quad \theta_{j0}(t) \geq 0,$$

where $\xi_{j0}(t)$ is the propensity to consume consumer goods, $\lambda_{j0}(t)$ the propensity to save, and $\theta_{j0}(t)$ is group j's propensity to consume fashion goods ($\theta_{C0}(t) = 0$).

2.6 Optimal household behavior

Maximizing the utility function subject to (9) yields

$$p_C(t)c_j(t) = \xi_j(t)\hat{y}_j(t), \quad s_j(t) = \lambda_j(t)\hat{y}_j(t), \quad p_F(t) f_j(t) = \theta_j(t)\hat{y}_j(t), \tag{10}$$

where

$$\xi_j(t) \equiv \rho_j(t)\xi_{j0}(t), \quad \lambda_j(t) \equiv \rho_j(t)\lambda_{j0}(t), \quad \theta_j(t) \equiv \rho_j(t)\theta_{j0}(t), \quad \rho_j(t) \equiv \frac{1}{\xi_{j0}(t) + \lambda_{j0}(t) + \theta_{j0}(t)}.$$

2.7 A simplified literature review on fashion dynamics and habit formation

As discussed by Zhang (2016), the modelling of $\theta_{S0}(t)$ and $\theta_{B0}(t)$ is influenced by Benhabib and Day (1981) and Giovinazzo and Naimzada (2015). The average conspicuous consumption in time t is defined as

$$\bar{f}(t) = n(t) f_B(t) + (1 - n(t)) f_S(t), \tag{11}$$

where

$$n(t) = \frac{\bar{N}_B(t)}{\bar{N}_B(t) + \bar{N}_S(t)}.$$

Each individual takes the average behavior of large populations $\bar{f}(t)$ as given when he makes decision. Following Giovinazzo and Naimzada, we assume that the snobs' and the bandwagoners' propensities to consume fashions as follows

$$\theta_j(t) = \Lambda_j\left(\bar{f}(t), t\right), \quad j = B, S. \tag{12}$$

We also generally assume that $\theta_S(t)$ ($\theta_B(t)$) is decreasing (increasing) in $\bar{f}(t)$.

2.8 Wealth accumulation

According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{13}$$

This equation states that the change in wealth is equal to saving minus dissaving.

2.9 Demand and supply of the three sectors

The demand and supply equilibrium for the consumer good sector is

$$c_C(t)\bar{N}_C(t) + c_S(t)\bar{N}_S(t) + c_B(t)\bar{N}_B(t) = F_s(t). \tag{14}$$

The demand and supply equilibrium for the fashion sector is

$$f_S(t)\bar{N}_S(t) + f_B(t)\bar{N}_B(t) = F_f(t). \tag{15}$$

As output of the capital good sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k(t)K(t) = F_i(t), \tag{16}$$

where

$$S(t) \equiv s_C(t)\bar{N}_C(t) + s_S(t)\bar{N}_S(t) + s_B(t)\bar{N}_B(t), \quad K(t) = \bar{k}_C(t)\bar{N}_C(t) + \bar{k}_S(t)\bar{N}_S(t) + \bar{k}_B(t)\bar{N}_B(t).$$

2.10 Capital being fully utilized

Total capital stock $K(t)$ is allocated to the three sectors

$$K_i(t) + K_s(t) + K_f(t) = K(t). \tag{17}$$

We completed the model. Zhang's model is a special case of the model in this study. The model is structurally general in the sense that some well-known models in economics can be considered as its special cases. For instance, if the population is homogeneous, all the parameters are constant, and there is no fashion industry, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is structurally similar to the Walrasian general equilibrium model (note that in our approach the population can be classified into any number of groups) if the wealth is fixed and depreciation is neglected.

3. The dynamics and its properties

The previous established a nonlinear dynamic system with wealth accumulation and different possible exogenous changes. It is difficult to provide general analytical properties of such a complicated system. Nevertheless, we now show that it is possible for us to conduct simulation to follow the motion of the dynamic system. The following lemma provides a computational procedure for calculating all the variables at any point in time. First, we introduce a new variable $z(t)$

$$z(t) \equiv \frac{r(t) + \delta_k(t)}{w(t)}.$$

3.1 Lemma

The motion of the economic system is determined by 3 differential equations with $z(t)$, $\bar{k}_B(t)$, $\bar{k}_S(t)$ and t as the variables

$$\begin{aligned} \dot{z}(t) &= \tilde{\Lambda}_z\left(z(t),\, \bar{k}_B(t),\, \bar{k}_S(t),\, t\right), \\ \dot{\bar{k}}_B(t) &= \tilde{\Lambda}_z\left(z(t),\, \bar{k}_B(t),\, \bar{k}_S(t),\, t\right), \\ \dot{\bar{k}}_S(t) &= \tilde{\Lambda}_z\left(z(t),\, \bar{k}_B(t),\, \bar{k}_S(t),\, t\right), \end{aligned} \tag{18}$$

in which $\tilde{\Lambda}_j(t)$ are unique functions of $z(t)$, $\bar{k}_B(t)$, $\bar{k}_S(t)$ and t defined in the appendix. At any point in time the other variables are unique functions of $z(t)$, $\bar{k}_B(t)$, $\bar{k}_S(t)$ and t determined by the following procedure: $r(t)$ and $w_j(t)$ by (A2) → $p_C(t)$ and $p_F(t)$ by (A3) → $\bar{f}(t)$ by (A12) → $\theta_j(t)$, $\rho_j(t)$, $\xi_j(t)$, and $\lambda_j(t)$ by their definitions → $\bar{k}_C(t)$ by (A14) → $\hat{y}_j(t)$ by (A9) → $K(t) = \sum_j \bar{k}_j(t)\bar{N}_j(t)$ → $c_j(t)$, $s_j(t)$ and $f_j(t)$ by (10) → $w(t) = w_1(t)/h_1(t)$ → $K_f(t)$ by (A5) → $N_s(t)$ by (A7) → $N_i(t)$ by (A6) → $K_j(t)$ by (A1) → $F_j(t)$ by the definitions.

We can easily follow the motion of the economic system. The preference changes are specified as follows

$$\theta_{B0}(t) = \bar{\theta}_B(t) + \tilde{\theta}_B(t)\,\bar{f}(t), \quad \theta_{S0}(t) = \bar{\theta}_S(t) - \tilde{\theta}_S(t)\,\bar{f}(t), \tag{19}$$

where $\bar{\theta}_j(t)$ and $\tilde{\theta}_j(t)$ are non-negative parameters. We specify

$$\xi_{B0}(t) = \bar{\xi}_B(t) - \omega_B(t)\tilde{\theta}_B(t)\bar{f}(t), \quad \lambda_{B0}(t) = \bar{\lambda}_B(t) - (1 - \omega_B(t))\tilde{\theta}_B(t)\bar{f}(t), \quad 0 \le \omega_B(t) \le 1,$$
$$\xi_{S0}(t) = \bar{\xi}_S(t) + \omega_S(t)\tilde{\theta}_S(t)\bar{f}(t), \quad \lambda_{S0}(t) = \bar{\lambda}_S(t) + (1 - \omega_S(t))\tilde{\theta}_S(t)\bar{f}(t), \quad 0 \le \omega_S(t) \le 1, \tag{20}$$

where $\omega_B(t)$ and $\omega_S(t)$ are "weight" parameters. For instance, if ω_B is small, it simply implies that a change in the propensity to consume fashion is associated with a relatively small change in the propensity to consume consumer goods and a relatively large change in the propensity to save. Equations (19) and (20) imply

$$\xi_j(t) \equiv \rho_j(t)\xi_{j0}(t), \quad \lambda_j(t) \equiv \rho_j(t)\lambda_{j0}(t), \quad \theta_j(t) = \rho_j(t)\theta_{j0}(t),$$
$$\rho_j(t) \equiv \frac{1}{\bar{\xi}_j(t) + \bar{\lambda}_j(t) + \bar{\theta}_j(t)}, \quad j = B, S. \tag{21}$$

Equations (11) and (10) imply

$$\bar{f}(t) = n(t)\frac{\rho_B(t)\theta_{B0}(t)\hat{y}_B(t)}{p_F(t)} + (1 - n(t))\frac{\rho_S(t)\theta_{S0}(t)\hat{y}_S(t)}{p_F(t)}, \tag{22}$$

where we use (21). Substitute (19) into (22)

$$\bar{f}(t) = \frac{n(t)\bar{\theta}_B(t)\rho_B(t)\hat{y}_B(t) + (1 - n(t))\bar{\theta}_S(t)\rho_S(t)\hat{y}_S(t)}{p_F(t) + (1 - n(t))\tilde{\theta}_S(t)\rho_S(t)\hat{y}_S(t) - n(t)\tilde{\theta}_B(t)\rho_B(t)\hat{y}_B(t)}. \tag{23}$$

The rest of this section summarizes the results in Zhang (2016), which correspond to the case that all the parameters are constant in the model of this study. In Zhang the parameters are specified as follows

$$A_i = 1, \ A_s = 1, \ A_f = 1.3, \ \alpha_i = 0.31, \ \alpha_s = 0.3, \ \alpha_f = 0.36, \ \delta_k = 0.07,$$
$$\begin{pmatrix} N_C \\ N_B \\ N_S \end{pmatrix} = \begin{pmatrix} 100 \\ 10 \\ 2 \end{pmatrix}, \ \begin{pmatrix} h_C \\ h_B \\ h_S \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \ \begin{pmatrix} \lambda_{C0} \\ \lambda_{B0} \\ \lambda_{S0} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.9 \\ 0.95 \end{pmatrix}, \ \begin{pmatrix} \xi_{C0} \\ \xi_{B0} \\ \xi_{S0} \end{pmatrix} = \begin{pmatrix} 0.15 \\ 0.11 \\ 0.1 \end{pmatrix}, \ \begin{pmatrix} \bar{\lambda}_{10} \\ \bar{\lambda}_{20} \\ \vec{\lambda}_{30} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.75 \\ 0.7 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\theta}_B \\ \bar{\theta}_S \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.01 \end{pmatrix}, \ \begin{pmatrix} \tilde{\theta}_B \\ \tilde{\theta}_S \end{pmatrix} = \begin{pmatrix} 0.001 \\ 0.001 \end{pmatrix}, \ \begin{pmatrix} \omega_B \\ \omega_S \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}. \tag{24}$$

The initial conditions as follows

$$z(0) = 0.06, \ \bar{k}_2(0) = 10, \ \bar{k}_3(0) = 4.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the national income is

$$Y(t) = F_i(t) + p_C(t)F_s(t) + p_F(t)F_f(t).$$

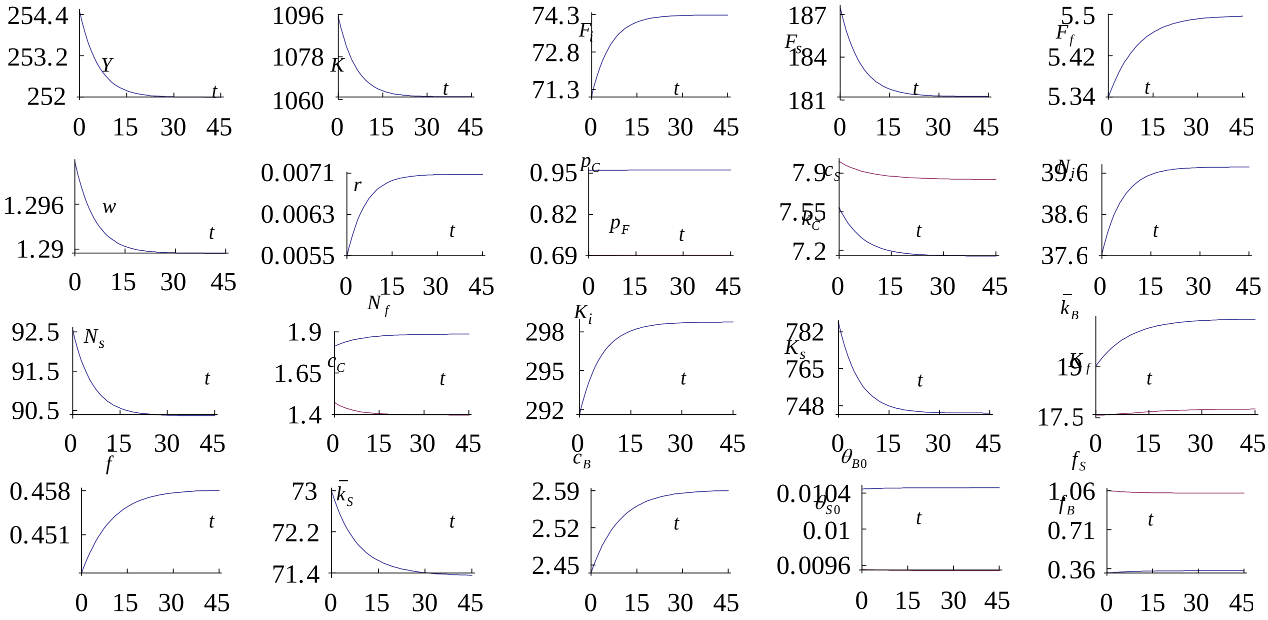

Figure 1: The Motion of the Economic System

The equilibrium values are listed as in (25)

$$Y = 252,\ K = 1061.17,\ F_i = 74.28,\ F_s = 181.24,\ F_f = 5.5,\ \bar{f} = 0.46,\ w_C = 1.29,\ w_B = 2.58,$$
$$w_S = 7.74,\ r = 0.007,\ p_C = 0.96,\ p_F = 0.69,\ N_i = 39.75,\ N_s = 90.37,\ N_f = 1.89,\ K_i = 298.8,$$
$$K_s = 744.64,\ K_f = 17.76,\ \bar{k}_C = 7.15,\ \bar{k}_B = 20.38,\ \bar{k}_S = 71.34,\ c_C = 1.4,\ c_B = 2.59,\ c_S = 7.84,$$
$$\theta_{B0} = 0.011,\ \theta_{S0} = 0.01,\ f_B = 0.343,\ f_S = 1.306. \quad (25)$$

The three eigenvalues are

$$\{-0.15,\ -0.12,\ -0.10\}.$$

The eigenvalues are real and negative. This guarantees the validity of exercising comparative dynamic analysis.

4. Comparative dynamic analysis

We simulated the motion of the national economy when all the parameters are constant. This study is interested in how the economic system behaves when it has time-dependent shocks. As the lemma provides the computational procedure to calibrate the motion of all the variables with any type of exogenous shocks, it is straightforward to examine effects of change in any parameter. We consider the parameters in (24) as the long-term average values. We make periodic perturbations around these long-term values. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter.

4.1 Periodic perturbations in the influence of the average fashion consumption on the bandwagoner's propensity to consume fashion

We now examine the effects of the following perturbations in the influence of the average fashion consumption on the bandwagoner's propensity to consume fashion good

$$\tilde{\theta}_B(t) = 0.001 + 0.001\sin(t).$$

The bandwagoner's propensity to consume fashion good is periodically affected by the average consumption. This implies that, for instance, if the snob consumes more fashion goods, the reaction strength of the bandwagoner is oscillatory. The simulation result is plotted in Figure 2. This oscillatory reaction causes business cycles. The amplitudes of changes are large in the fashion industry. The bandwagoner's consumption level of consumer good changes greatly.

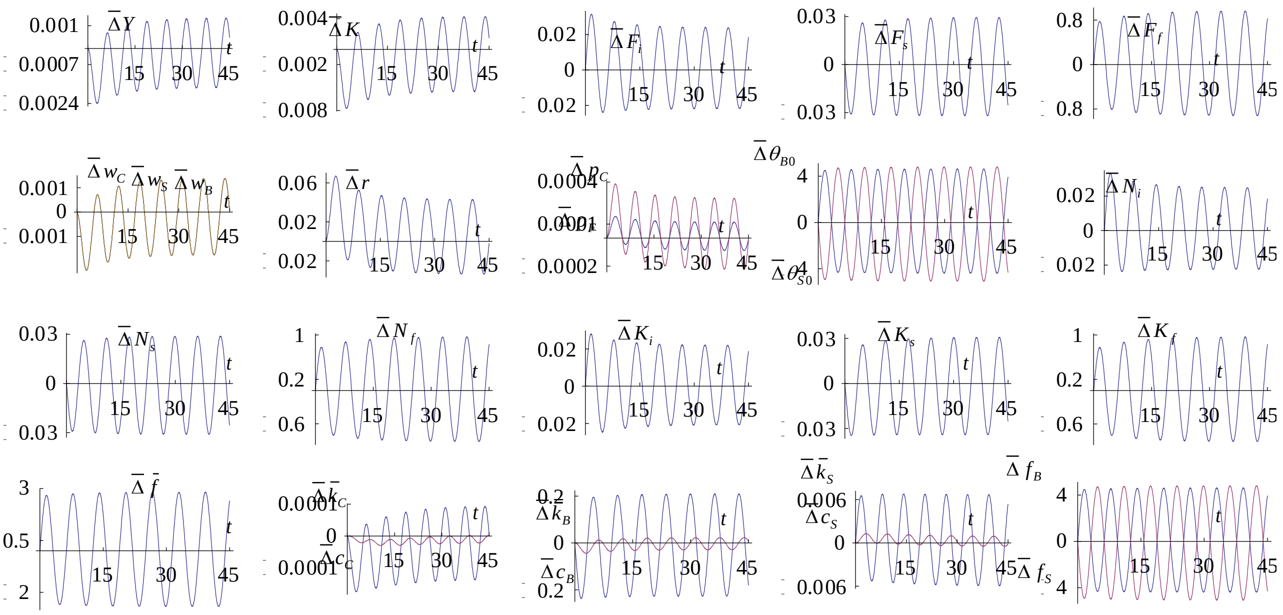

Figure 2. Periodic perturbations in the influence of the average fashion consumption on the bandwagoner's propensity to consume fashion

4.2 Periodic perturbations in the influence of the average fashion consumption on the snob's propensity to consume fashion

We now examine the effects of the following periodic perturbations in the influence of the average fashion consumption on the snob's propensity to consume fashion:

$$\tilde{\theta}_S(t) = 0.001 + 0.001\sin(t).$$

The simulation result is plotted in Figure 3. We see that in comparison to the impact on the bandwagoner the impact of perturbations on the snob has much weaker impact on the fashion industry.

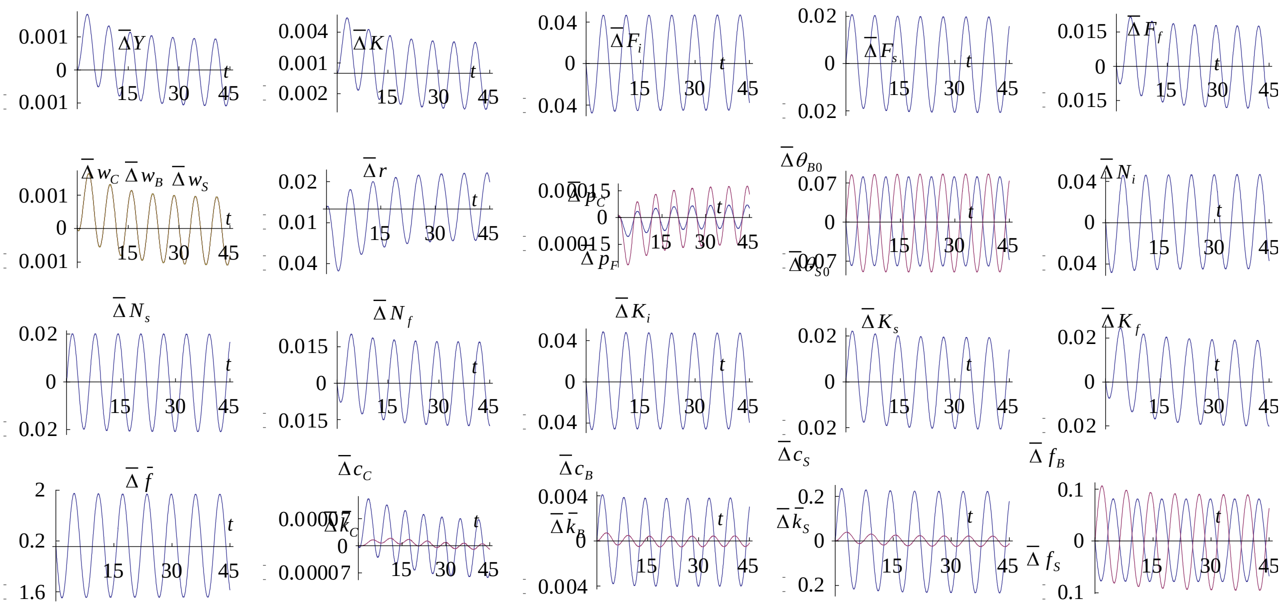

Figure 3. Periodic Perturbations in the Influence of the Average Fashion Consumption on the Snob's Propensity to consume fashion

4.3 Perturbations on the bandwagoner's propensity to consume fashion

We now examine the effects of the following fluctuations of the intercept in the function of the bandwagoner's propensity to consume fashion goods

$$\bar{\theta}_B(t) = 0.01 + 0.01\sin(t).$$

The perturbations in the parameter implies periodic shocks in the propensity to consume fashion goods. The economic structure experiences periodic changes. The amplitude of the national wealth is much greater than that of the national output.

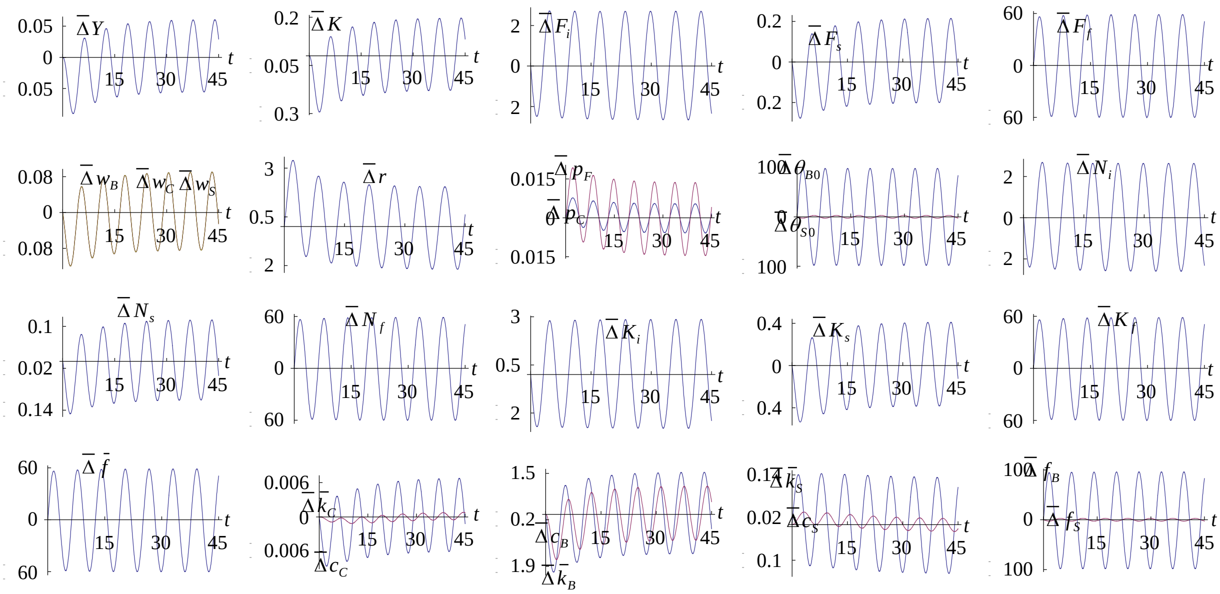

Figure 4. Perturbations on the Bandwagoner's Propensity to Consume Fashion

4.4 Periodic perturbations in the bandwagoner's human capital

We now allow the bandwagoner's human capital to be oscillatory as follows

$$h_B(t) = 2 + 0.2\sin(t).$$

The result is plotted in Figure 5. The perturbations cause business cycles. The national income fluctuates more greatly than the national capital.

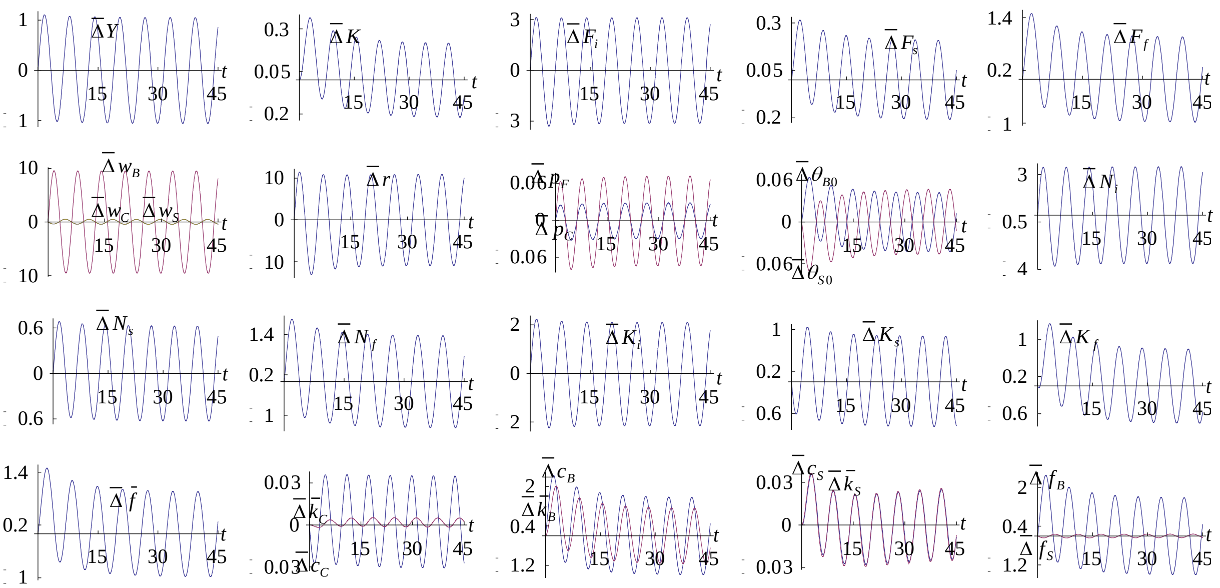

Figure 5. Periodic Perturbations in the Bandwagoner's Human Capital

4.5 Perturbations in the bandwagoner's population

We now examine the effects of the following perturbations in the bandwagoner's population

$$N_B(t) = 10 + 0.5\sin(t).$$

The result is plotted in Figure 6. The perturbations cause periodic changes in macroeconomic variables and have slight impact on the microeconomic variables. The greatest amplitude of oscillations occurs in the fashion industry.

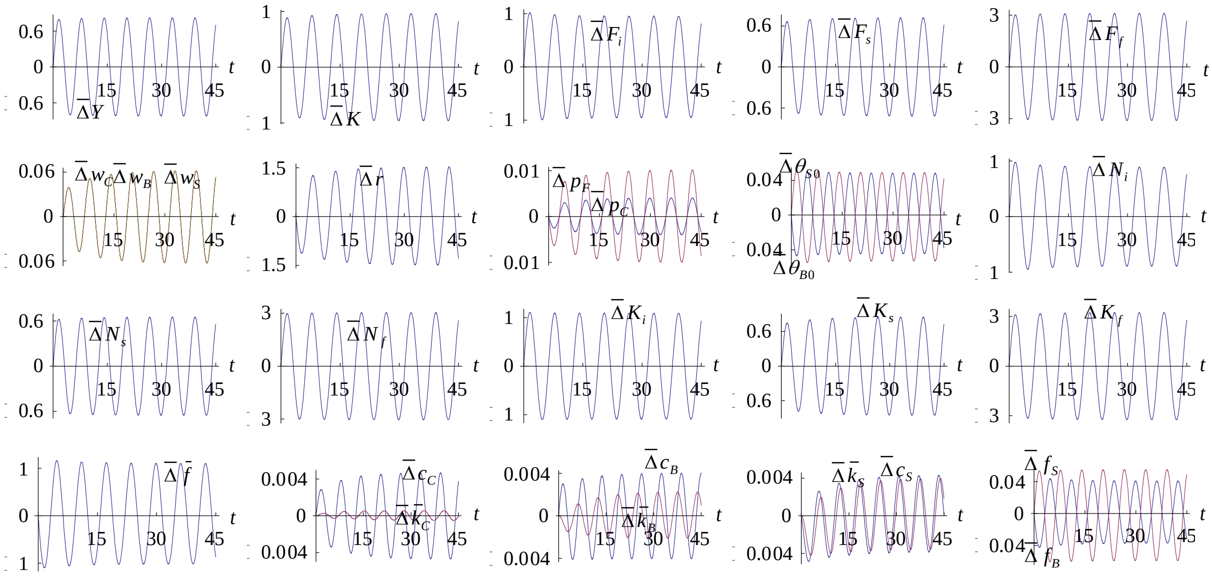

Figure 6. A Rise in the Bandwagoner's Population

4.6 Perturbations in the snob's propensity to save

We now examine the effects of perturbations in the intercept of the function of the snob's propensity to save as follows:

$$\bar{\lambda}_S(t) = 0.95 + 0.02\sin(t).$$

The result is plotted in Figure 7.

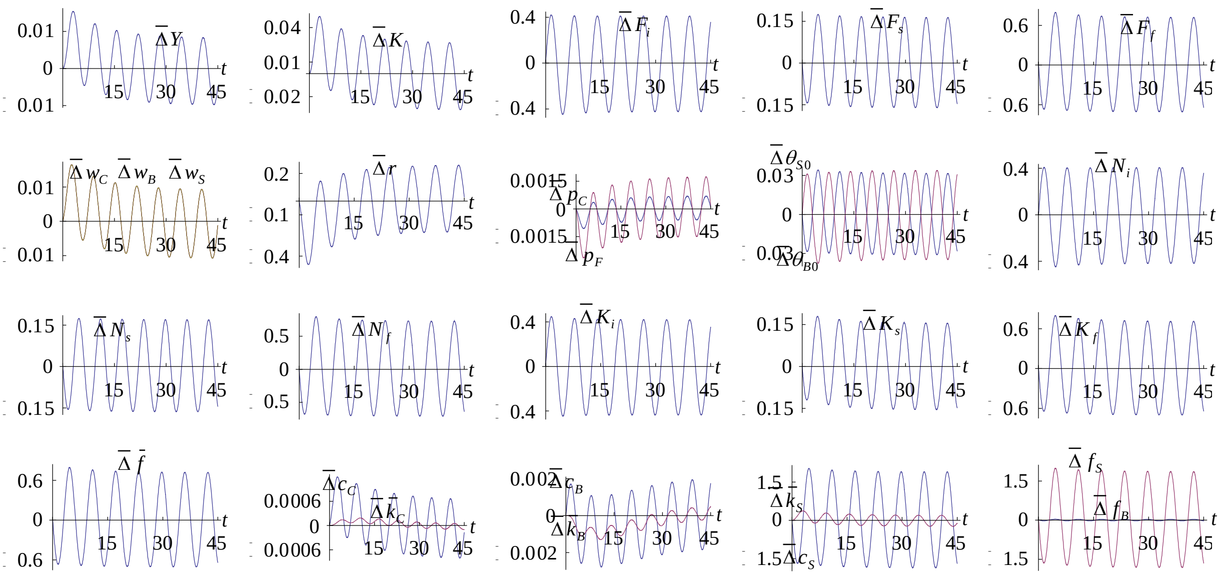

Figure 7. A Rise in The Snob's Propensity to Save

5. Concluding remarks

This paper generalized the dynamic growth model with interactions between fashion, economic growth and income and wealth distribution. The model deals with economic growth of heterogeneous households with economic structure. The model introduced fashion into neoclassical growth model. The original model was based on some ideas in the literature of economics of fashion. This study generalized Zhang's model by making all the time-independent parameters as time-dependent parameters. We simulated the motion of the economic system. We carried out comparative dynamic analysis with regard to periodic perturbations in the snob's propensity to consume fashion goods, the bandwagoner's propensity to consume fashion goods, the bandwagoner's human capital, the bandwagoner's population, and the snob's propensity to save. We examined how exogenous period changes in these parameters lead to business cycles. We may generalize and extend model in different directions. One important direction is to introduce taxation into the general dynamic equilibrium model with fashion. There are fashions in fields of arts, sciences, politics, academics, business, food, clothing, morality and entertainment in different forms and changes in different speeds. To analyze dynamics of fashion with endogenous knowledge and wealth is a challenging issue in theoretical economics.

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Appendix: Proving the lemma

From (4) and (7), we get

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s} = \frac{N_f}{\bar{\beta}_f K_f}, \tag{A1}$$

where $\bar{\beta}_j \equiv \beta_m / \alpha_m$. Substituting (A1) into (4) and (5) yields

$$r = \alpha_r z^{\beta_i} - \delta_k , \quad w_j = \alpha_w h_j z^{-\alpha_i} , \tag{A2}$$

where

$$\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i} , \quad \alpha_w = \beta_i A_i \bar{\beta}_i^{-\alpha_i} .$$

Hence, the rate of interest and the wage rates are determined as functions of z. Equations (6) and (7) imply

$$p_m = \frac{\bar{\beta}_m^{\alpha_m} z^{\alpha_m} w}{\beta_m A_m} . \tag{A3}$$

Substituting $p_C c_j = \xi_j \hat{y}_j$ into (18) yields

$$\xi_C \bar{N}_C \hat{y}_C + \xi_B \bar{N}_B \hat{y}_B + \xi_S \bar{N}_S \hat{y}_S = \frac{w N_s}{\beta_s} , \tag{A4}$$

where we use $p_C F_s = w N_s / \beta_s$. Similarly for fashion goods we have

$$\theta_B \bar{N}_B \hat{y}_B + \theta_S \bar{N}_S \hat{y}_S = \frac{w N_f}{\beta_f} . \tag{A5}$$

Substitute (A4) and (A5) into (2)

$$N_i = \hat{N} - \hat{n} \hat{y}_C , \tag{A6}$$

where

$$\hat{N} \equiv N - \frac{(\theta_B \beta_f + \xi_B \beta_s) \bar{N}_B \hat{y}_B + (\theta_S \beta_f + \xi_S \beta_s) \bar{N}_S \hat{y}_S}{w} , \quad \hat{n} \equiv \frac{\xi_C \beta_s \bar{N}_C}{w} .$$

Insert (A1) in (17)

$$z K = \frac{N_i}{\bar{\beta}_i} + \frac{N_s}{\bar{\beta}_s} + \frac{N_f}{\bar{\beta}_f} . \tag{A7}$$

Insert (A4)-(A6) in (A7)

$$z K = \hat{N}_1 + \hat{n}_1 \hat{y}_C , \tag{A8}$$

where

$$\hat{N}_1 \equiv \frac{\hat{N}}{\bar{\beta}_i} + \frac{\xi_B \beta_s \bar{N}_B \hat{y}_B + \xi_S \beta_s \bar{N}_S \hat{y}_S}{\bar{\beta}_s w} + \frac{\theta_B \beta_f \bar{N}_B \hat{y}_B + \theta_S \beta_f \bar{N}_S \hat{y}_S}{\bar{\beta}_f w} , \quad \hat{n}_1 \equiv \frac{\xi_C \beta_s \bar{N}_C}{\bar{\beta}_s w} - \frac{\hat{n}}{\bar{\beta}_i} .$$

The definitions of $\hat{y}_j$ imply

$$\hat{y}_j = (1 + r) \bar{k}_j + w_j . \tag{A9}$$

Insert (A9) in (A8)

$$zK = \hat{N}_1 + \hat{n}_1 w_C + (1+r)\hat{n}_1 \bar{k}_C . \tag{A10}$$

Equations (10) and (11) imply

$$\bar{f} = \frac{n\theta_B(\bar{f})\hat{y}_B}{p_F} + \frac{(1-n)\theta_S(\bar{f})\hat{y}_S}{p_F}. \tag{A11}$$

Denote solutions of (A11) with $\bar{f}$ as follows

$$\bar{f} = f_0(z, \bar{k}_B, \bar{k}_S, t). \tag{A12}$$

It is straightforward to see that we can treat θ_j, ρ_j, ξ_j and λ_j as functions of z, $\bar{k}_B$, and $\bar{k}_S$. Equations (16) and (A10) imply

$$\hat{N}_1 + \hat{n}_1 w_C + (1+r)\hat{n}_1 \bar{k}_C = z\bar{k}_C \bar{N}_C + z\bar{k}_S \bar{N}_S + z\bar{k}_B \bar{N}_B . \tag{A13}$$

Solve (A13) with $\bar{k}_C$ as the variable

$$\bar{k}_C = \Lambda(z, \bar{k}_B, \bar{k}_S, t) \equiv \frac{z\bar{k}_S \bar{N}_S + z\bar{k}_B \bar{N}_B - \hat{N}_1 - \hat{n}_1 w_C}{(1+r)\hat{n}_1 - z\bar{N}_C}. \tag{A14}$$

It is straightforward to confirm that all the variables can be expressed as functions of z, $\bar{k}_B$, and $\bar{k}_S$, by the following procedure: r and w_j by (A2) → p_C and p_F by (A3) → $\bar{f}$ by (A12) → θ_j, ρ_j, ξ_j and λ_j by their definitions → $\bar{k}_C$ by (A14) → $\hat{y}_j$ by (A9) → $K = \sum_j \bar{k}_j \bar{N}_j$ → c_j, s_j and f_j by (10) → $w = w_1 / h_1$ → K_f by (A5) → N_s by (A7)→ N_i by (A6) → K_j by (A1) → F_j by the definitions. From this procedure, (A12) and (17), we have

$$\dot{\bar{k}}_C = \tilde{\Lambda}_C(z, \bar{k}_B, \bar{k}_S, t) \equiv \lambda_C \hat{y}_C - \bar{k}_C , \tag{A15}$$

$$\dot{\bar{k}}_j = \tilde{\Lambda}_j(z, \bar{k}_B, \bar{k}_S, t) \equiv \lambda_j \hat{y}_j - \bar{k}_j , \quad j = B, S. \tag{A16}$$

Taking derivatives of equation (A14) with respect to t yields

$$\dot{\bar{k}}_C = \frac{\partial \Lambda}{\partial t} + \frac{\partial \Lambda}{\partial z}\dot{z} + \tilde{\Lambda}_B \frac{\partial \Lambda}{\partial \bar{k}_B} + \tilde{\Lambda}_C \frac{\partial \Lambda}{\partial \bar{k}_C}. \tag{A17}$$

Here we don't give explicit expressions as they are tedious. Equaling the two right-hand sides in (A15) and (A17) implies

$$\dot{z} = \tilde{\Lambda}_z(z, \bar{k}_B, \bar{k}_S, t) \equiv \left(\tilde{\Lambda}_C - \frac{\partial \Lambda}{\partial t} - \tilde{\Lambda}_B \frac{\partial \Lambda}{\partial \bar{k}_B} - \tilde{\Lambda}_C \frac{\partial \Lambda}{\partial \bar{k}_C}\right)\left(\frac{\partial \Lambda}{\partial z}\right)^{-1}. \tag{A18}$$

The differential equations system (A16) and (A18) has 3 variables, z, $\bar{k}_B$ and $\bar{k}_S$ and consists of 3 differential equations. In summary, we proved the lemma.